Chapter 1

1.1 Suppose the input to an amplifier is $x_a(t) = \sin(2\pi F_0 t)$ and the steady-state output is

$$y_a(t) = 100\sin(2\pi F_0 t + \phi_1) - 2\sin(4\pi F_0 t + \phi_2) + \cos(6\pi F_0 t + \phi_3)$$

(a) Is the amplifier a linear system or is it a nonlinear system?
(b) What is the gain of the amplifier?
(c) Find the average power of the output signal.
(d) What is the total harmonic distortion of the amplifier?

Solution

(a) The amplifier is *nonlinear* because the steady-state output contains harmonics.
(b) From (1.1.2), the amplifier gain is $K = 100$.
(c) From (1.2.4), the output power is

$$\begin{aligned} P_y &= \frac{d_0^2}{4} + \frac{1}{2}\left(d_1^2 + d_+2^2 + d_3^2\right) \\ &= .5(100^2 + 2^2 + 1) \\ &= 5002.5 \end{aligned}$$

(d) From (1.2.5)

$$\begin{aligned} \text{THD} &= \frac{100(P_y - d_1^2/2)}{P_y} \\ &= \frac{100(5002.5 - 5000)}{5002.5} \\ &= .05\% \end{aligned}$$

√ 1.2 Consider the following *signum* function that returns the sign of its argument.

$$\text{sgn}(t) \triangleq \begin{cases} 1 & , \quad t > 0 \\ 0 & , \quad t = 0 \\ -1 & , \quad t < 0 \end{cases}$$

(a) Using Appendix 1, find the magnitude spectrum.

(b) Find the phase spectrum.

Solution

(a) From Table A2 in Appendix 1

$$X_a(f) = \frac{1}{j\pi f}$$

Thus the magnitude spectrum is

$$\begin{aligned} A_a(f) &= |X_a(f)| \\ &= \frac{1}{|j\pi f|} \\ &= \frac{1}{\pi |f|} \end{aligned}$$

(b) The phase spectrum is

$$\begin{aligned} \phi_a(f) &= \angle X_a(f) \\ &= -\angle j\pi f \\ &= -\text{sgn}(f)\left(\frac{\pi}{2}\right) \end{aligned}$$

1.3 Parseval's identity states that a signal and its spectrum are related in the following way.

$$\int_{-\infty}^{\infty} |x_a(t)|^2 dt = \int_{-\infty}^{\infty} |X_a(f)|^2 df$$

Use Parseval's identity to compute the following integral.

$$J = \int_{-\infty}^{\infty} \text{sinc}^2(2Bt) dt$$

Solution

From Table A2 in Appendix 1 if

$$x_a(t) = \text{sinc}(2Bt)$$

then

$$X_a(f) = \frac{\mu_a(f+B) - \mu_a(f-B)}{2B}$$

Thus by Parseval's identity

$$\begin{aligned} J &= \int_{-\infty}^{\infty} \sin^2(2Bt)dt \\ &= \int_{-\infty}^{\infty} |x_a(t)|^2 dt \\ &= \int_{-\infty}^{\infty} |X_a(f)|^2 df \\ &= \frac{1}{2B}\int_{-B}^{B} df \\ &= 1 \end{aligned}$$

1.4 Consider the causal exponential signal

$$x_a(t) = \exp(-ct)\mu_a(t)$$

(a) Using Appendix 1, find the magnitude spectrum.
(b) Find the phase spectrum
(c) Sketch the magnitude and phase spectra when $c = 1$.

Solution

(a) From Table A2 in Appendix 1

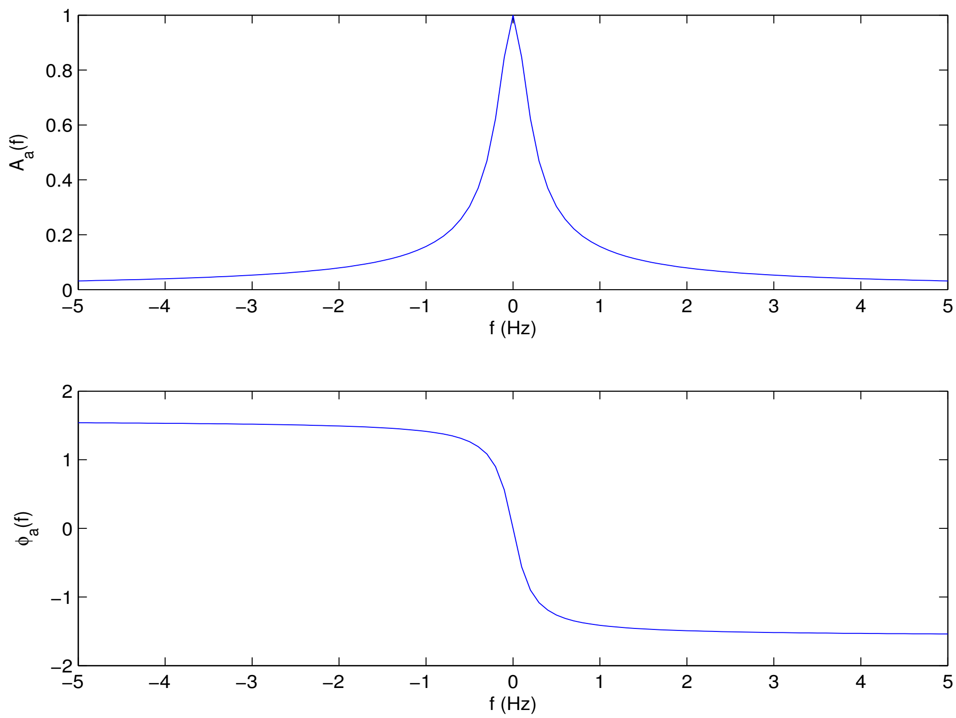
$$X_a(f) = \frac{1}{c + j2\pi f}$$

Thus the magnitude spectrum is

$$\begin{aligned} A_a(f) &= |X_a(f)| \\ &= \frac{1}{|c + j2\pi f|} \\ &= \frac{1}{\sqrt{c^2 + (2\pi f)^2}} \end{aligned}$$

(b) The phase spectrum is

$$\begin{aligned} A_a(f) &= |X_a(f)| \\ &= \angle 1 - \angle(c + j2\pi f) \\ &= -\tan^{-1}\left(\frac{2\pi f}{c}\right) \end{aligned}$$



Problem 1.4 (c) Magnitude and Phase Spectra, $c = 1$

1.5 If a real analog signal $x_a(t)$ is square integrable, then the *energy* that the signal contains within the frequency band $[F_0, F_1]$ where $F_1 \geq F_0 \geq 0$ can be computed as follows.

$$E(F_0, F_1) = 2\int_{F_0}^{F_1} |X_a(f)|^2 df$$

Consider the following double exponential signal with $c > 0$.

$$x_a(t) = \exp(-c|t|)$$

(a) Find the total energy, $E(0, \infty)$.

(b) Find the percentage of the total energy that lies in the frequency range $[0, 2]$ Hz.

Solution

(a) From Table A2 in Appendix 1

$$X_a(f) = \frac{2c}{c^2 + 4\pi^2 f^2}$$

Thus the total energy of $x_a(t)$ is

$$\begin{aligned} E(0,\infty) &= 2\int_0^\infty |X_a(f)|^2 df \\ &= 2\int_0^\infty \left[\frac{2c}{c^2+4\pi^2 f^2}\right] df \\ &= \frac{4c}{2\pi c}\tan^{-1}\left(\frac{2\pi f}{c}\right)\Big|_0^\infty \\ &= \frac{2}{\pi}\left(\frac{\pi}{2}\right) \\ &= 1 \end{aligned}$$

(b) Using part (a), the percentage of the total energy that lies in the frequency range $[0, 2]$ Hz is

$$\begin{aligned} p &= \frac{100E(0,2)}{E(0,\infty)} \\ &= 100E(0,2) \\ &= \frac{200}{\pi}\tan^{-1}\left(\frac{2\pi f}{c}\right)\Big|_0^2 \\ &= \frac{200}{\pi}\tan^{-1}\left(\frac{4\pi}{c}\right)\% \end{aligned}$$

1.6 Let $x_a(t)$ be a periodic signal with period T_0. The *average power* of $x_a(t)$ can be defined as follows.

$$P_x = \frac{1}{T_0}\int_0^{T_0} |x_a(t)|^2 dt$$

Find the average power of the following periodic continuous-time signals.

(a) $x_a(t) = \cos(2\pi F_0 t)$

(b) $x_a(t) = c$

(c) A periodic train of pulses of amplitude a, duration T, and period T_0.

Solution

(a) Using Appendix 2,

$$\begin{aligned} P_x &= F_0 \int_0^{1/F_0} \cos^2(2\pi F_0 t) dt \\ &= \frac{F_0}{2} \int_0^{1/F_0} [1 + \cos(4\pi F_0 t)] dt \\ &= \frac{1}{2} \end{aligned}$$

(b)

$$\begin{aligned} P_x &= \frac{1}{T_0} \int_0^{T_0} c^2 dt \\ &= c^2 \end{aligned}$$

(c)

$$\begin{aligned} P_x &= \frac{1}{T_0} \int_0^{T} a^2 dt \\ &= \frac{a^2 T}{T_0} \end{aligned}$$

1.7 Consider the following discrete-time signal where the samples are represented using N bits.

$$x(k) = \exp(-ckT)\mu(k)$$

(a) How many bits are needed to ensure that the quantization level is less than .001?

(b) Suppose $N = 8$ bits. What is the average power of the quantization noise?

Solution

(a) For $k \geq 0$, the signal ranges over $0 \leq x(k) \leq 1$. Thus $x_{\min} = 0$ and $x_{\max} = 1$ and from (1.2.3) the quantization level is

$$q = \frac{1}{2^N}$$

Setting $q = .001$ yields

$$\frac{1}{2^N} = \frac{1}{1000}$$

Taking the log of both sides, $-N\ln(2) = -\ln(1000)$ or

$$\begin{aligned} N &= \text{ceil}\left[\frac{\ln(1000)}{\ln(2)}\right] \\ &= \text{ceil}(9.966) \\ &= 10 \text{ bits} \end{aligned}$$

(b) From (1.2.8) the average power of the quantization noise using $N = 8$ bits is

$$\begin{aligned} E[e^2] &= \frac{q^2}{12} \\ &= \frac{1}{12(2^N)^2} \\ &= 1.271 \times 10^{-6} \end{aligned}$$

1.8 Show that the spectrum of a causal signal $x_a(t)$ can be obtained from the Laplace transform $X_a(s)$ by replacing s by $j2\pi f$. Is this also true for noncausal signals?

Solution

For a causal signal $x_a(t)$, the one-sided Laplace transform can be extended to a two-sided transform without changing the result.

$$X_a(s) = \int_{-\infty}^{\infty} x_a(t)\exp(-st)dt$$

If s is now replaced by $j2\pi f$, this reduces to the Fourier transform $X_a(f)$ in (1.2.16). Thus the spectrum of a causal signal can be obtained from the Laplace transform as follows.

$$X_a(f) = X_a(s)|_{s=j2\pi f} \quad \text{if} \quad x_a(t) = 0 \text{ for } t < 0$$

This is *not* true for a noncausal signal where $x_a(t) \neq 0$ for $t < 0$.

1.9 Consider the following periodic signal.

$$x_a(t) = 1 + \cos(10\pi t)$$

(a) Compute the magnitude spectrum of $x_a(t)$.

(b) Suppose $x_a(t)$ is sampled with a sampling frequency of $f_s = 8$ Hz. Sketch the magnitude spectrum of $x_a(t)$ and the sampled signal, $\hat{x}_a(t)$.

(c) Does aliasing occur when $x_a(t)$ is sampled at the rate $f_s = 8$ Hz? What is the folding frequency in this case?

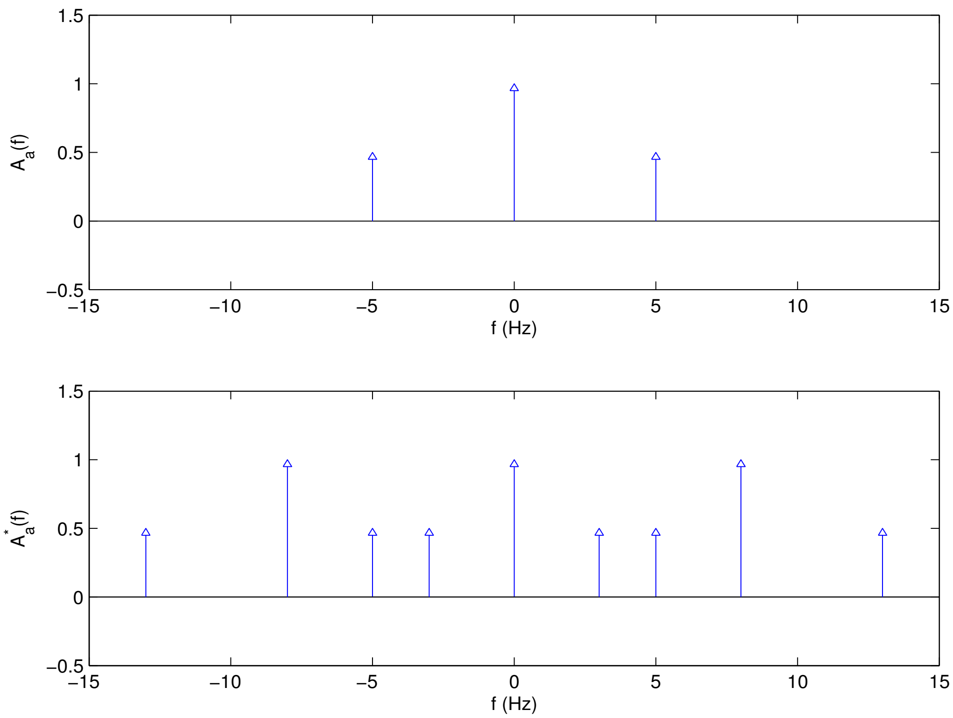
(d) Find a range of values for the sampling interval T which ensures that aliasing will not occur.

(e) Assuming $f_s = 8$ Hz, find an alternative lower-frequency signal, $x_b(t)$, that has the same set of samples as $x_a(t)$.

Solution

(a) From the linearity property and Table A2 in Appendix 1

$$X_a(f) = \delta_a(f) + \frac{\delta_a(f+5) + \delta_a(f-5)}{2}$$



Problem 1.9 (b) Magnitude Spectra

(c) *Yes*, aliasing does occur (see sketch). The folding frequency is

$$\begin{aligned} f_d &= \frac{f_s}{2} \\ &= 4 \text{ Hz} \end{aligned}$$

(d) The signal $x_a(t)$ is bandlimited to 5 Hz. From Proposition 1.1, to avoid aliasing, the sampling rate must satisfy $f_s > 10$. Thus $1/T > 10$ or

$$0 < T < .1 \text{ sec}$$

(e) Using the trigonometric identities from Appendix 2 with $f_s = 8$

$$
\begin{aligned}
x(k) &= 1+\cos(10\pi kT) \\
&= 1+\cos(1.25\pi k) \\
&= 1+\cos(2\pi k - .75\pi k) \\
&= 1+\cos(2\pi k)\cos(.75\pi k)+\sin(2\pi k)\sin(.75\pi k) \\
&= 1+\cos(.75\pi k) \\
&= 1+\cos(6\pi k/8) \\
&= 1+\cos(6\pi kT)
\end{aligned}
$$

Thus an alternative lower-frequency signal with the same set of samples is

$$x_b(t) = 1+\cos(6\pi t)$$

√ 1.10 Consider the following bandlimited signal.

$$x_a(t) = \sin(4\pi t)[1+\cos^2(2\pi t)]$$

(a) Using the trigonometric identities in Appendix 2, find the maximum frequency present in $x_a(t)$.

(b) For what range of values for the sampling interval T can this signal be reconstructed from its samples?

Solution

(a) From Appendix 2

$$
\begin{aligned}
x_a(t) &= \sin(4\pi t)+\sin(4\pi t)\cos^2(2\pi t) \\
&= \sin(4\pi t)+.5\sin(4\pi t)[1+\cos(4\pi t)] \\
&= \sin(4\pi t)+.5\sin(4\pi t)+.5\sin(4\pi t)\cos(4\pi t) \\
&= \sin(4\pi t)+.5\sin(4\pi t)+.25\sin(8\pi t)
\end{aligned}
$$

Thus the highest frequency present in $x_a(t)$ is $F_0 = 4$ Hz.

(b) From Proposition 1.1, to avoid aliasing $f_s > 8$ Hz. Thus

$$0 < T < .125 \text{ sec}$$

1.11 It is not uncommon for students to casually restate the sampling theorem in the following way: "A signal must be sampled at twice the highest frequency present to avoid aliasing". Interesting enough, this informal formulation is not quite correct. To verify this, consider the following simple signal.

$$x_a(t) = \sin(2\pi t)$$

(a) Find the magnitude spectrum of $x_a(t)$, and verify that the highest frequency present is $F_0 = 1$ Hz.

(b) Suppose $x_a(t)$ is sampled at the rate $f_s = 2$ Hz. Sketch the magnitude spectrum of $x_a(t)$ and the sampled signal, $\hat{x}_a(t)$. Do the replicated spectra overlap?

(c) Compute the samples $x(k) = x_a(kT)$ using the sampling rate $f_s = 2$ Hz. Is it possible to reconstruct $x_a(t)$ from $x(k)$ using the reconstruction formula in Proposition 1.2 in this instance?

(d) Restate the sampling theorem in terms of the highest frequency present, but this time correctly.

Solution

(a) From Table A2 in Appendix 2

$$X_a(f) = \frac{j[\delta_a(f+1) - \delta_a(f-1)]}{2}$$

Thus the magnitude spectrum of $x_a(t)$ is

$$A_a(f) = \frac{\delta_a(f+1) + \delta_a(f-1)}{2}$$

Clearly, the highest frequency present is $F_0 = 1$ Hz. See sketch.

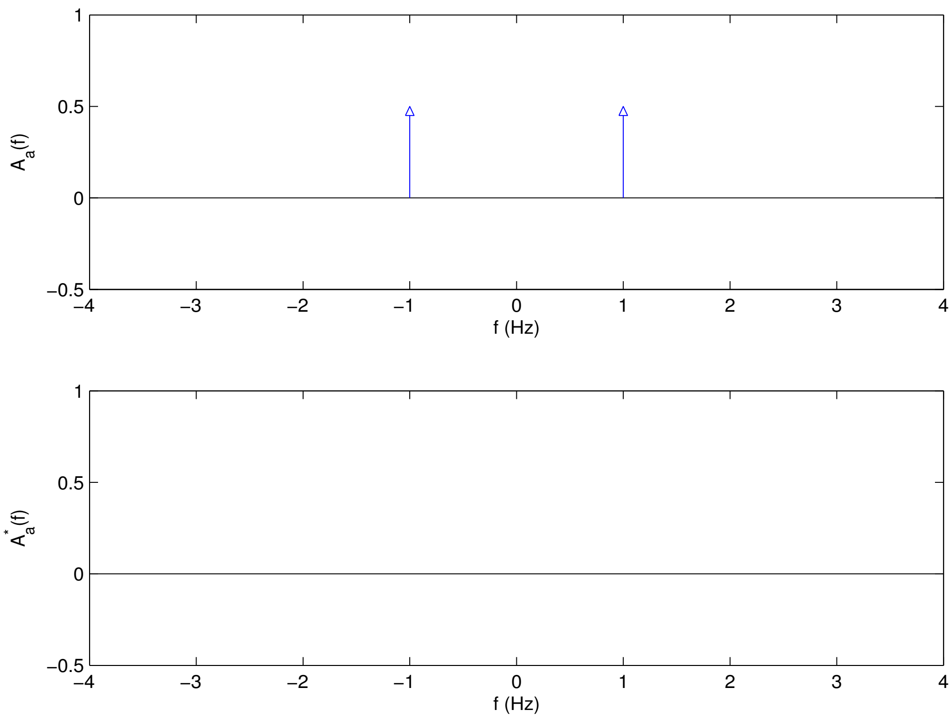
(b) Yes, the replicated spectra do overlap (see sketch). In this instance, the overlapping spectra cancel one another.

(c) When $f_s = 2$, the samples are

$$\begin{aligned} x(k) &= \sin(2\pi k T) \\ &= \sin(\pi k) \\ &= 0 \end{aligned}$$

No, it is not possible to reconstruct $x_a(t)$ from these samples using Proposition 1.2.

(d) A signal must be sampled at a rate that is *higher* than twice the highest frequency present to avoid aliasing.



Problem 1.11 (b) Magnitude Spectra

1.12 Why is it not possible to physically construct an ideal lowpass filter? Use the impulse response, $h_a(t)$, to explain your answer.

Solution

From Example 1.4, an ideal lowpass filter with gain one and cutoff frequency B has the following impulse response

$$h_a(t) = 2B\text{sinc}(2Bt)$$

Therefore $h_a(t) \neq 0$ for $t < 0$. This makes the impulse response a noncausal signal and the system that produced it a noncausal system. Noncausal systems are not physically realizable because the system would have to anticipate the input (an impulse at time $t = 0$) and respond to it before it occurred.

1.13 There are special circumstances where it is possible to reconstruct a signal from its samples even when the sampling rate is less than twice the bandwidth. To see this, consider a signal $x_a(t)$ whose spectrum $X_a(f)$ has a hole in it as shown in Figure 1.46.

(a) What is the bandwidth of the signal $x_a(t)$ whose spectrum is shown in Figure 1.46? The pulses are of radius 100 Hz.

(b) Suppose the sampling rate is $f_s = 750$ Hz. Sketch the spectrum of the sampled signal $\hat{x}_a(t)$.

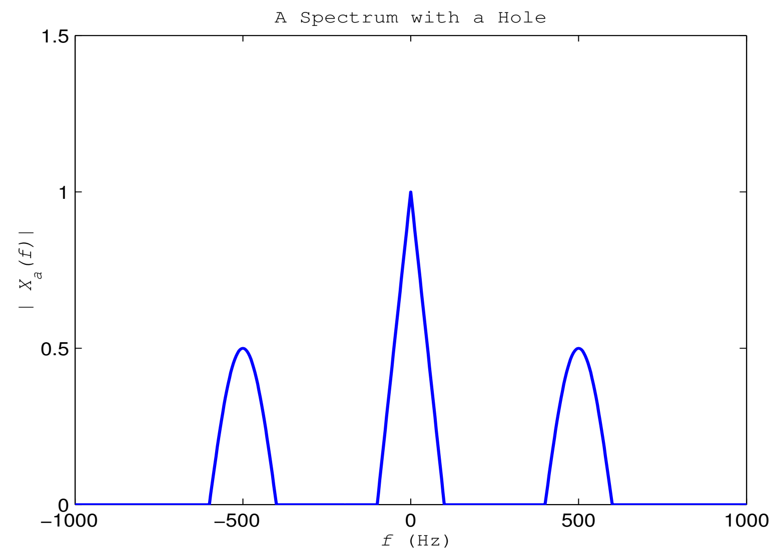
(c) Show that $x_a(t)$ can be reconstructed from $\hat{x}_a(t)$ by finding an idealized reconstruction filter with input $\hat{x}_a(t)$ and output $x_a(t)$. Sketch the magnitude response of the reconstruction filter.

(d) For what range of sampling frequencies below $2f_s$ can the signal be reconstructed from the samples using the type of reconstruction filter from part (c)?

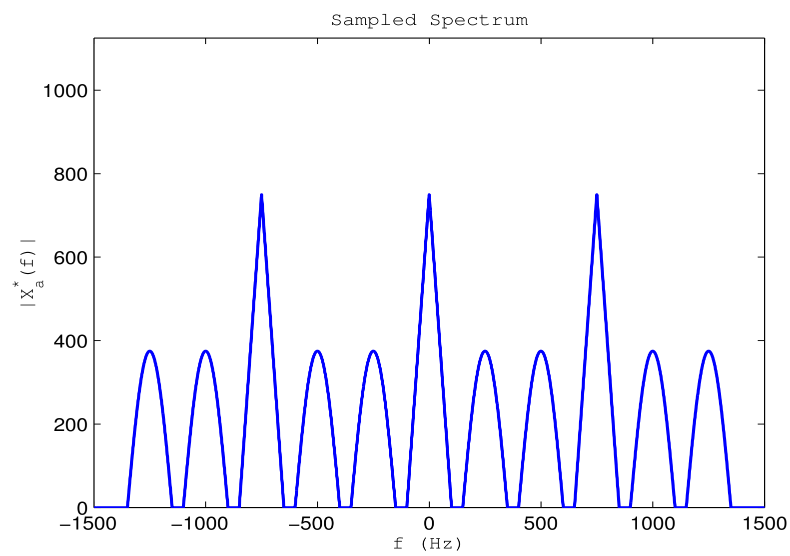
Solution

(a) From inspection of Figure 1.46, the bandwidth of $x_a(t)$ is $B = 600$ Hz.

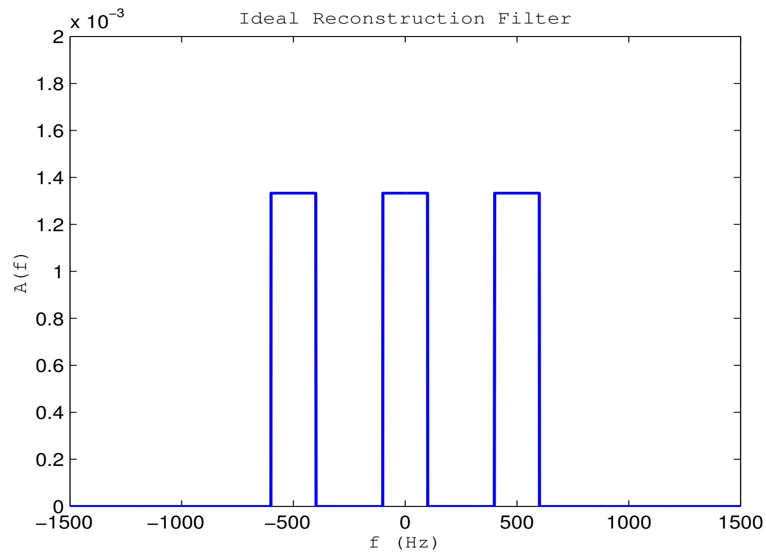
(d) From inspection of the solution to part (c), the signal can be reconstructed from the samples (no overlap of the spectra) for $700 < f_s < 800$ Hz.



Problem 1.46 A Signal Whose Spectrum Has a Hole in It



Problem 1.13b (b) Magnitude Spectrum of Sampled Signal



Problem 1.13c (c) Magnitude Response of Ideal Reconstruction Filter

1.14 Consider the problem of using an anti-aliasing filter as shown in Figure 1.47. Suppose the anti-aliasing filter is a lowpass Butterworth filter of order $n = 4$ with cutoff frequency $F_c = 2$ kHz.

(a) Find a lower bound f_L on the sampling frequency that ensures that the aliasing error is reduced by a factor of at least .005.

(b) The lower bound f_L represents oversampling by what factor?

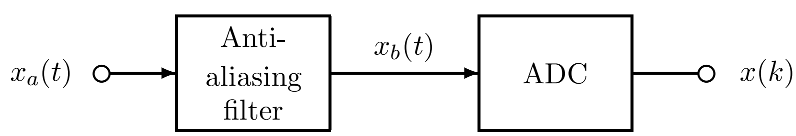


Figure 1.47 Preprocessing with an Anti-Aliasing Filter

Solution

(a) Suppose $f_s = 2\alpha F_c$ for some $\alpha > 1$. Using (1.5.1) and evaluating $H_a(f)$ at the folding frequency $f_d = f_s/2$ we have

$$\frac{1}{\sqrt{1+\alpha^8}} = .005$$

Squaring both sides and taking reciprocals

$$1 + \alpha^8 = 40000$$

Solving for α

$$\begin{aligned} \alpha &= 39999^{1/8} \\ &= 3.761 \end{aligned}$$

Thus the lower bound on the cutoff frequency is

$$\begin{aligned} f_L &= 2\alpha F_c \\ &= 2(3.761)2000 \\ &= 15.044 \text{ kHz} \end{aligned}$$

(b) This represents oversampling by a factor of factor $\alpha = 3.761$.

1.15 Show that the transfer function of a linear continuous-time system is the Laplace transform of the impulse response.

Solution

Let $y_a(t)$ be the impulse response. Using Definition 1.8 and Table A4 in Appendix 1

$$\begin{aligned} L\{y_a(t)\} &= Y_a(s) \\ &= H_a(s)X_a(s) \\ &= H_a(s)L\{\delta_a(t)\} \\ &= H_a(s) \end{aligned}$$

1.16 A bipolar DAC can be constructed from a unipolar DAC by inserting an operational amplifier at the output as shown in Figure 1.48. Note that the unipolar N-bit DAC uses a reference voltage of $2V_R$, rather than $-V_r$ as in Figure 1.36. This means that the unipolar DAC output is $-2y_a$ where y_a is given in (1.6.4). Analysis of the operational amplifier section of the circuit reveals that the bipolar DAC output is then

$$z_a = 2y_a - V_r$$

(a) Find the range of values for z_a.

(b) Suppose the binary input is $b = b_{N-1}b_{N-2}\cdots b_0$. For what value of b is $z_a = 0$?

(c) What is the quantization level of this bipolar DAC?

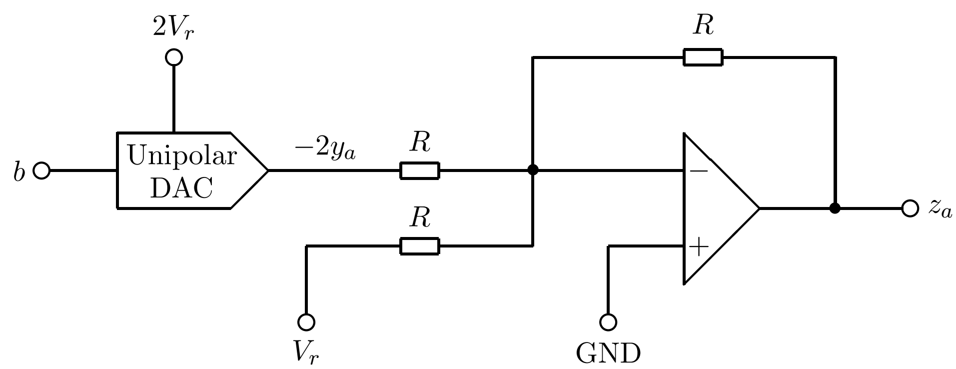


Figure 1.48 A Bipolar N-bit DAC

Solution

(a) From (1.6.5) we have $0 \leq y_a \leq (2^N - 1)V_r/2^N$. When $y_a = 0$, this yields $z_a = -V_r$. The upper limit of z_a is

$$\begin{aligned} z_a &\geq \frac{2(2^N-1)V_r}{2^N} - V_r \\ &= \frac{[2(2^N-1) - 2^N]V_r}{2^N} \\ &= \frac{(2^N-2)V_r}{2^N} \end{aligned}$$

Thus the range of values for the bipolar DAC output is

$$-V_r \leq z_a \leq \left(\frac{2^N - 2}{2^N}\right) V_r$$

(b) If $b = 10 \cdots 0$, then from (1.6.4) and (1.6.1) we have

$$\begin{aligned} y_a &= \left(\frac{V_r}{2^N}\right) x \\ &= \left(\frac{V_r}{2^N}\right) \sum_{k=0}^{N-1} b_k 2^k \\ &= \left(\frac{V_r}{2^N}\right) 2^{N-1} \\ &= \frac{V_r}{2} \end{aligned}$$

The bipolar DAC output is then

$$\begin{aligned} z_a &= 2y_a - V_r \\ &= 0 \end{aligned}$$

(c) From (1.2.3), the quantization level of a bipolar DAC with output $-V_r \leq z_a < V_r$ is

$$\begin{aligned} q &= \frac{V_r - (-V_r)}{2^N} \\ &= \frac{V_r}{2^{N-1}} \end{aligned}$$

√ 1.17 Suppose a bipolar ADC is used with a precision of $N = 12$ bits and a reference voltage of $V_r = 10$ volts.

(a) What is the quantization level q?

(b) What is the maximum value of the magnitude of the quantization noise, assuming the ADC input-output characteristics is offset by $q/2$ as in Figure 1.35?

(c) What is the average power of the quantization noise?

Solution

(a) From (1.6.7)

$$\begin{aligned} q &= \frac{V_r}{2^{N-1}} \\ &= \frac{10}{2^{11}} \\ &= .0049 \end{aligned}$$

(b) The maximum quantization error, assuming rounding, is

$$\begin{aligned} E_{\max} &= \frac{q}{2} \\ &= .0024 \end{aligned}$$

(c) From (1.2.8), the average power of the quantization noise is

$$\begin{aligned} E[e^2] &= \frac{q^2}{12} \\ &= 1.9868 \times 10^{-6} \end{aligned}$$

1.18 Suppose an 8-bit bipolar successive approximation ADC has reference voltage $V_r = 10$ volts.

(a) If the analog input is $x_a = -3.941$ volts, find the successive approximations by filling in the entries in Table 1.10.

(b) If the clock rate is $f_{\text{clock}} = 200$ kHz, what is the sampling rate of this ADC?

(c) Find the quantization level of this ADC.

(d) Find the average power of the quantization noise.

Table 1.10 Successive Approximations

k	b_{n-k}	u_k	y_k
0			
1			
2			
3			
4			
5			
6			
7			

Solution

(a) Applying Alg. 1.1, the successive approximations are as follows

Table 1.8 Successive Approximations

k	b_{n-k}	u_k	y_k
0	0	0	-10.0000
1	1	1	-5.0000
2	0	0	-5.0000
3	0	0	-5.0000
4	1	1	-4.3750
5	1	1	-4.0625
6	0	0	-4.0625
7	1	1	-3.9844

(b) Since there are $N = 8$ bits, the successive approximation sampling rate is

$$\begin{aligned} f_s &= \frac{f_{\text{clock}}}{N} \\ &= \frac{2 \times 10^5}{8} \\ &= 25 \text{ kHz} \end{aligned}$$

(c) Using (1.6.7), the quantization level of this bipolar ADC is

$$\begin{aligned} q &= \frac{10}{2^7} \\ &= .0781 \end{aligned}$$

(d) Using (1.2.8) the average power of the quantization noise is

$$
\begin{aligned}
E[e^2] &= \frac{q^2}{12} \\
&= 5.083 \times 10^{-4}
\end{aligned}
$$

1.19 An alternative to the R-$2R$ ladder DAC is the weighted-resistor DAC shown in Figure 1.49 for the case $N = 4$. Here the switch controlled by bit b_k is open when $b_k = 0$ and closed when $b_k = 1$. Recall that the decimal equivalent of the binary input b is as follows.

$$
x = \sum_{k=0}^{N-1} b_k 2^k
$$

(a) Show that the current through the kth branch of an N-bit weighted-resistor DAC is

$$
I_k = \frac{-V_r b_k}{2^{N-k} R} \quad , \quad 0 \le k < N
$$

(b) Show that the DAC output voltage is

$$
y_a = \left(\frac{V_r}{2^N}\right) x
$$

(c) Find the range of output values for this DAC.

(d) Is this DAC unipolar, or is it bipolar?

(e) Find the quantization level of this DAC.

Solution

(a) The kth branch (starting from the right) has resistance $2^{N-k}R$. For an ideal op amp, the principle of the virtual short circuit says that the voltage drop between the noninverting terminal (+) and the inverting terminal(−) is zero. Thus $V = 0$. Applying Ohm's law, current through the kth branch is

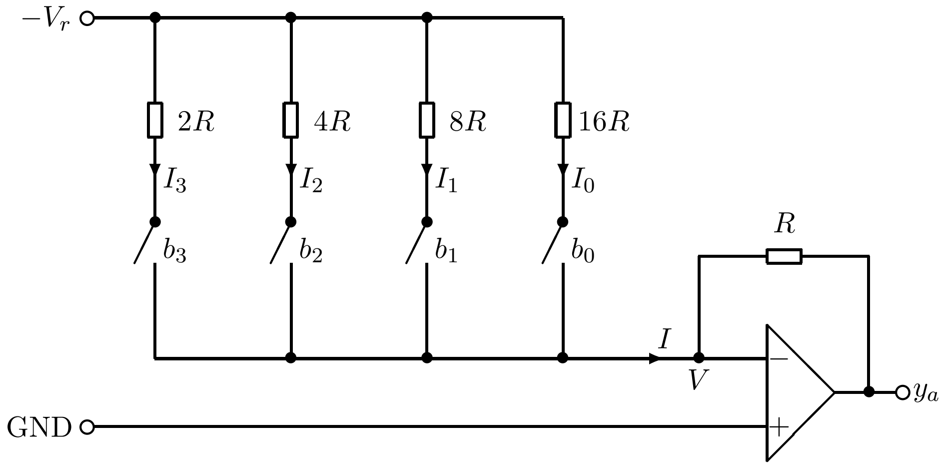


Figure 1.49 A Four-Bit Weighted-resistor DAC

$$
\begin{aligned}
I_k &= \frac{(-V_r - V)b_k}{2^{N-k}R} \\
&= \frac{-V_r b_k}{2^{N-k}R} \qquad , \qquad 0 \le k < N
\end{aligned}
$$

(b) For an ideal op amp, there is no current flowing into the inverting input (infinite input impedance). Consequently, using $V = 0$ and I_k from part (a),

$$
\begin{aligned}
y_a &= V - RI \\
&= -RI \\
&= -R \sum_{i=0}^{N-1} I_k \\
&= -R \sum_{i=0}^{N-1} \frac{-V_r b_k}{2^{N-k}R} \\
&= V_r \sum_{i=0}^{N-1} b_k 2^{k-N} \\
&= \left(\frac{V_r}{2^N}\right) \sum_{i=0}^{N-1} b_k 2^k \\
&= \left(\frac{V_r}{2^N}\right) x
\end{aligned}
$$

(c) Since x ranges from 0 to 2^{N-1}, it follows from part (b) that

$$0 \leq y_a \leq \left(\frac{2^{N-1}}{2^N}\right) V_r$$

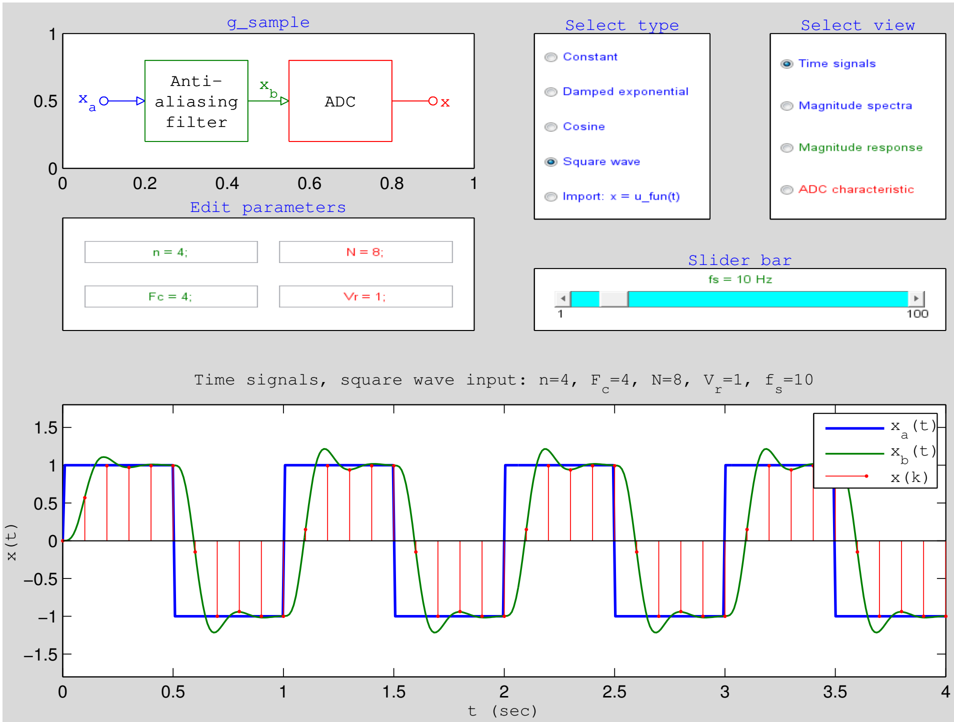
(d) Since $y_a \geq 0$, this is a *unipolar* DAC.

(e) For the unipolar DAC, $0 \leq y_a < V_r$. Thus from (1.2.3), the quantization level is

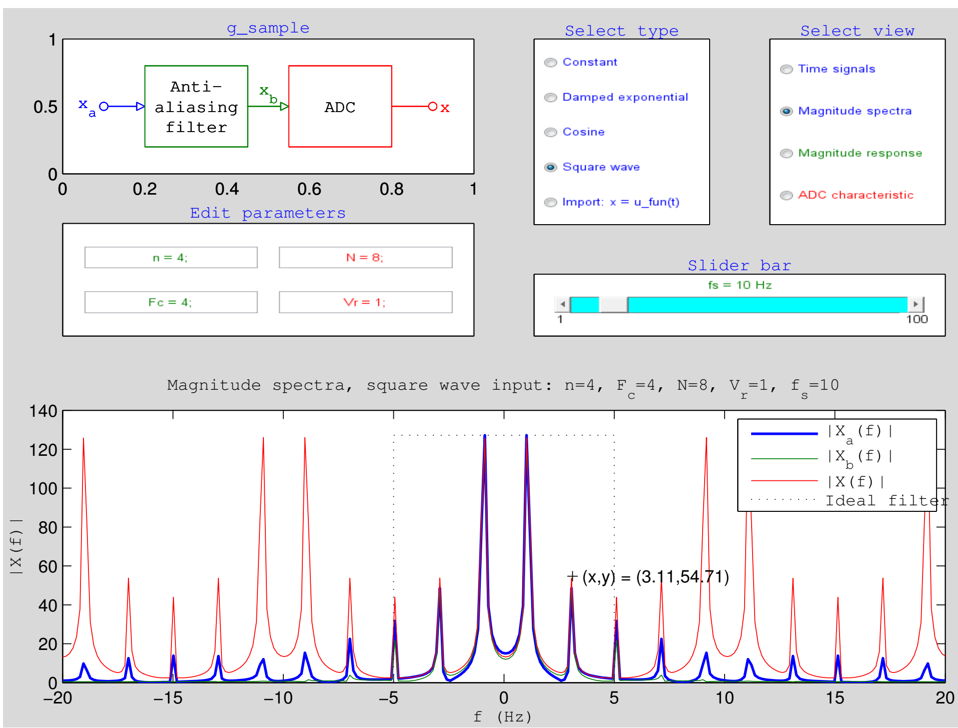
$$\begin{aligned} q &= \frac{V_r - 0}{2^N} \\ &= \frac{V_r}{2^N} \end{aligned}$$

1.20 Use GUI module *g_sample* to plot the time signals and magnitude spectra of the square wave using $fs = 10$ Hz. On the magnitude spectra plot, use the Caliper option to display the amplitude and frequency of the third harmonic. Are there even harmonics present the square wave?

Solution



Problem 1.20 (a)



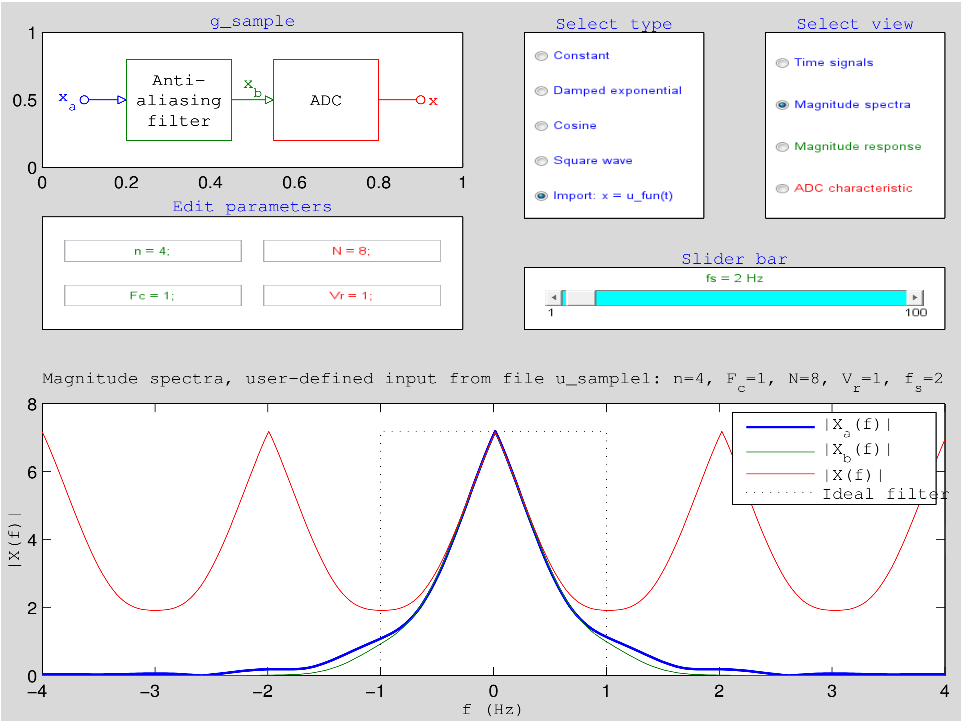
Problem 1.20 (b) There are no even harmonics

√ 1.21 Use GUI module *g_sample* to Import the signal in the file *u_sample1* and plot its magnitude spectra. Set $F_c = 1$ and do the following two cases. For which ones is there noticeable aliasing?

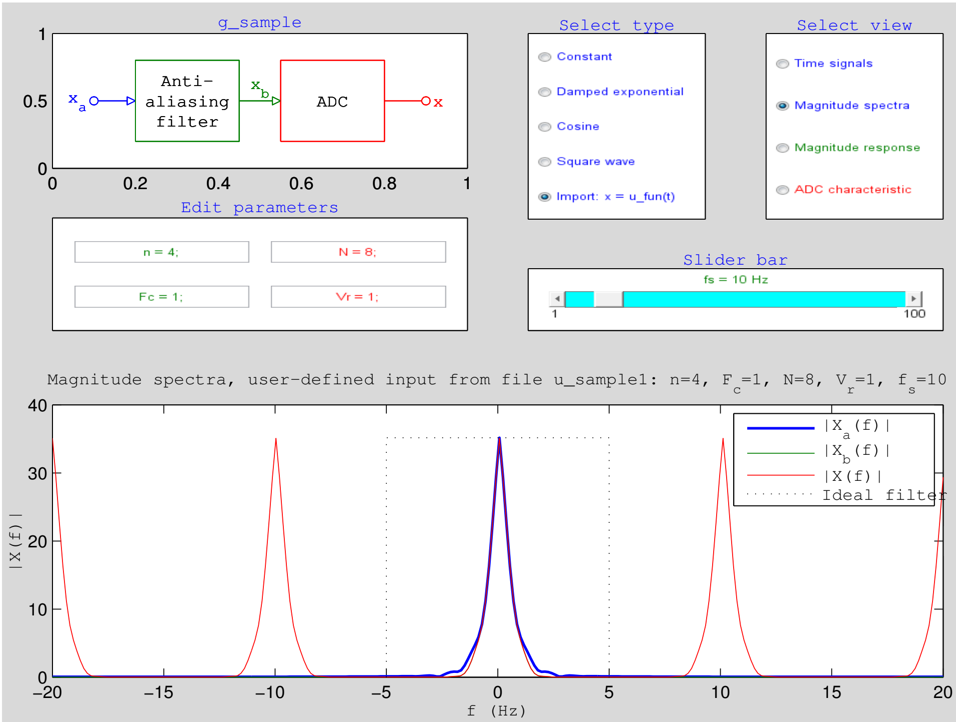
(a) $fs = 2$ Hz

(b) $fs = 10$ Hz

Solution



Problem 1.21 (a) Significant aliasing, $f_s = 2$ Hz



Problem 1.21 (b) No significant aliasing, $f_s = 10$ Hz

1.22 Consider the following exponentially damped sine wave with $c = 1$ and $F_0 = 1$.

$$x_a(t) = \exp(-ct)\sin(2\pi F_0 t)\mu_a(t)$$

(a) Write a MATLAB function called *u_sample2* that returns the value $x_a(t)$.

(b) Use the User-Defined option in GUI module *g_sample* to sample this signal at $f_s = 12$ Hz. Plot the time signals.

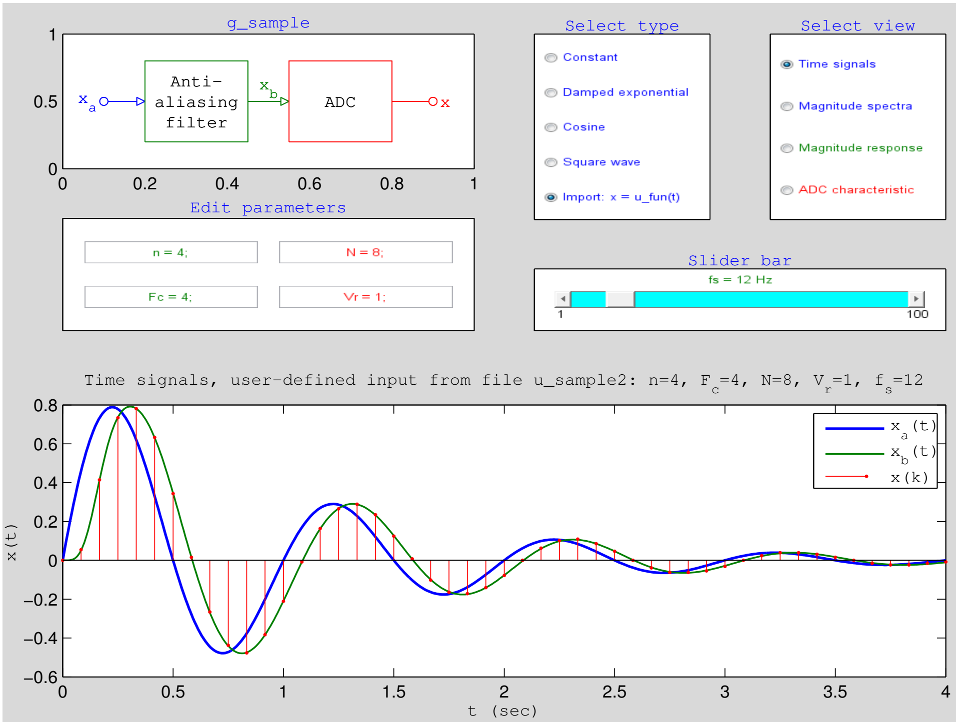
(c) Adjust the sampling rate to $f_s = 4$ Hz and set the cutoff frequency to $F_c = 2$ Hz. Plot the magnitude spectra.

Solution

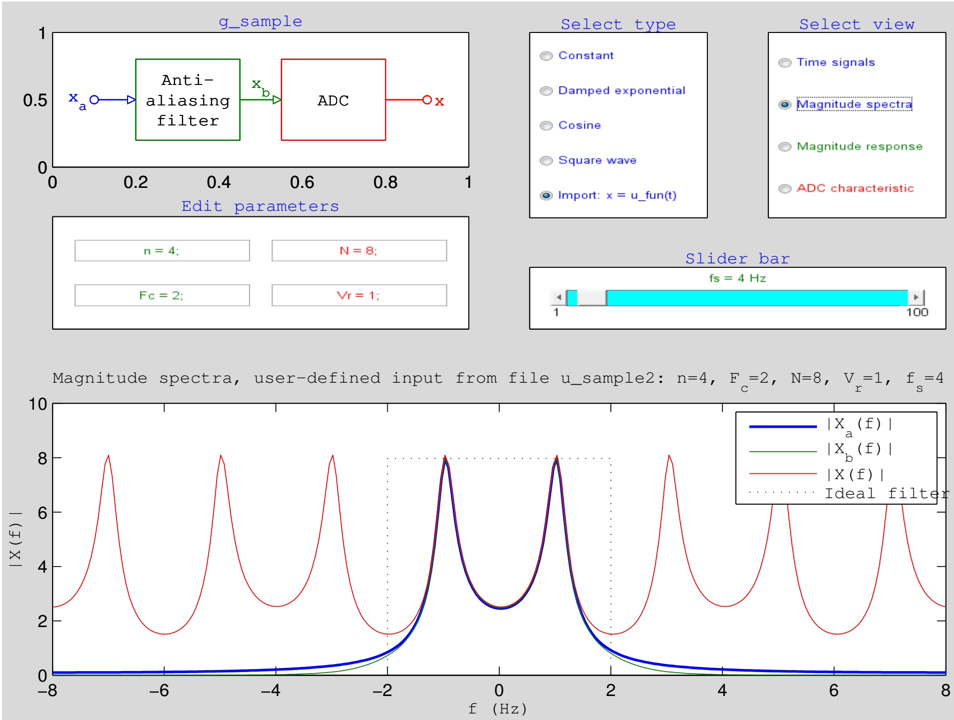
(a) Write a MATLAB function called *u_sample2* that returns the value $x_a(t)$.

```
function y = u_sample2 (t)

%U_SAMPLE2: User file for problem 1.22
%
% Usage: y = u_sample2 (t);
```

```
%
% Inputs: t  = vector of input times
%
% Outputs: y = vector of samples of analog signal evaluated at t

y = exp(-t) .* sin(2*pi*t);
```



Problem 1.22 (b) Time Plots



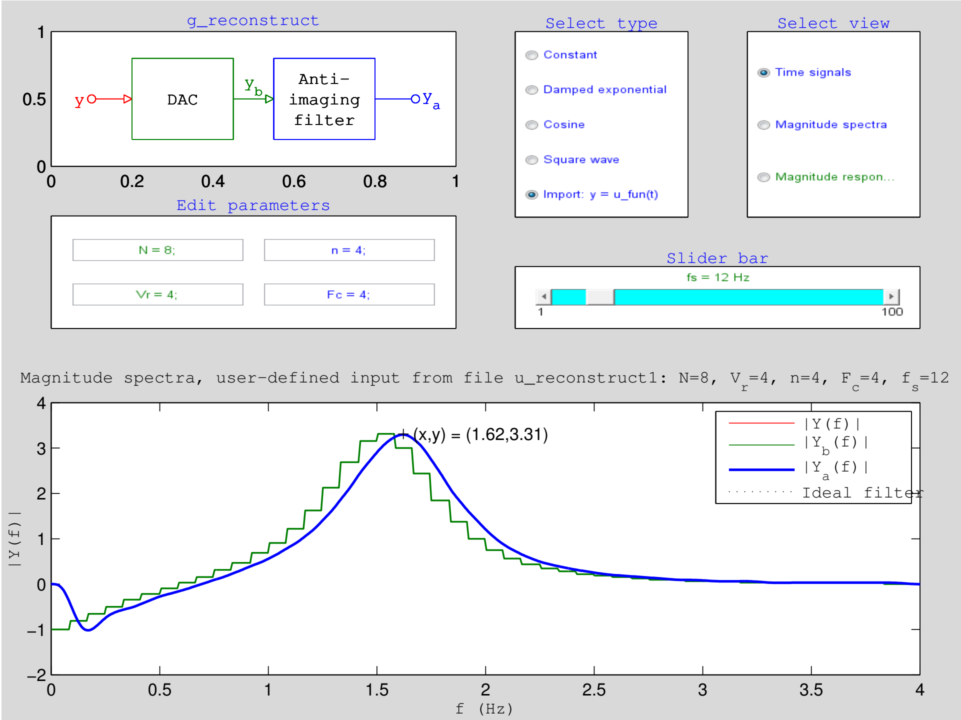
Problem 1.22 (c) Spectra

√ 1.23 Use GUI module *g_reconstruct* to Import the signal in the file, *u_reconstruct1*. Adjust fs to 12 Hz and set $V_r = 4$.

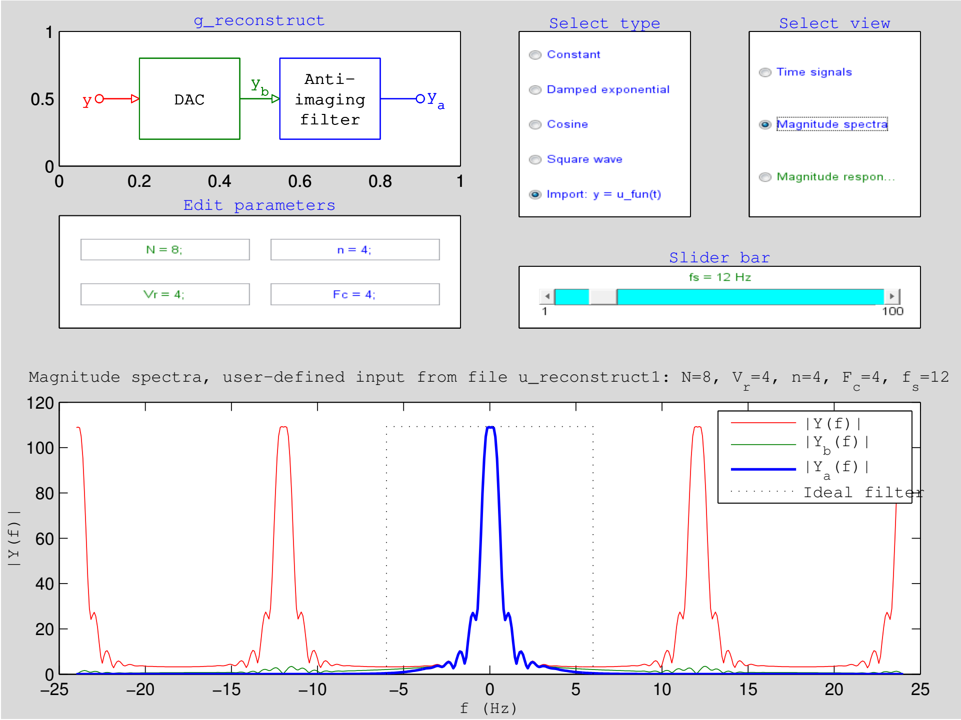
(a) Plot the time signals, and use the Caliper option to identify the amplitude and time of the peak output.

(b) Plot the magnitude spectra.

Solution



Problem 1.23 (a)



Problem 1.23 (b)

1.24 Consider the exponentially damped sine wave in problem 1.22.

(a) Write a MATLAB function that returns the value $x_a(t)$.

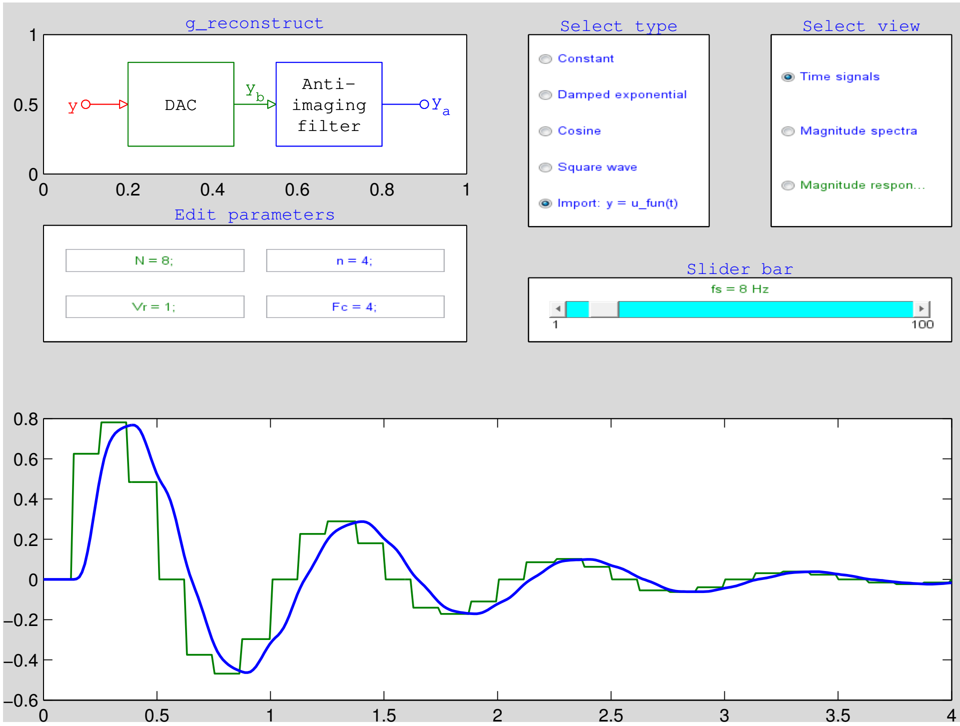
(b) Use the User-Defined option in GUI module *g_reconstruct* to sample this signal at $f_s = 8$ Hz. Plot the time signals.

(c) Adjust the sampling rate to $f_s = 4$ Hz and set $F_c = 2$ Hz. Plot the magnitude spectra.

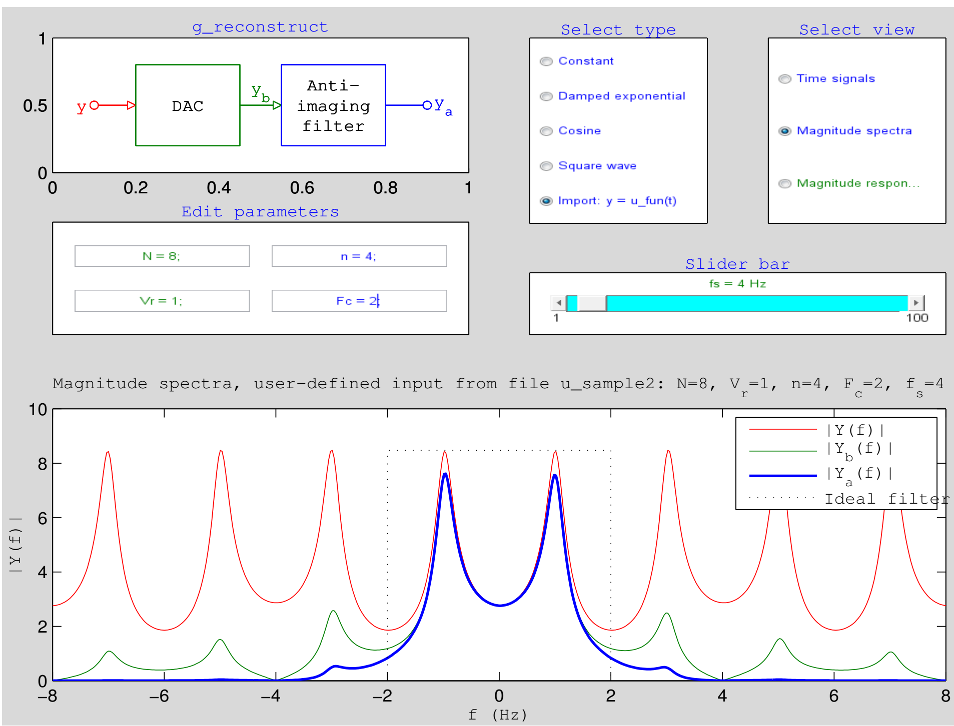
Solution

(a) Write a MATLAB function that returns the value $x_a(t)$.

```
function y = u_sample2 (t)

%U_SAMPLE2: User file for problem 1.24
%
% Usage: y = u_sample2 (t);
%
% Inputs: t  = vector of input times
%
% Outputs: y = vector of samples of analog signal evaluated at t

y = exp(-t) .* sin(2*pi*t);
```



Problem 1.24 (b)



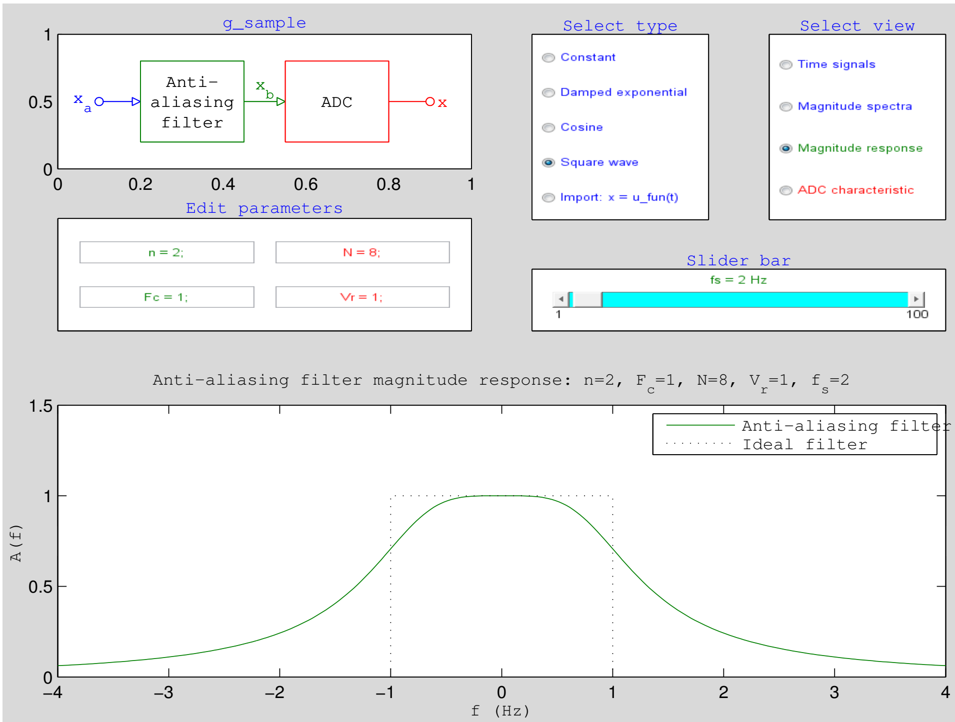
Problem 1.24 (c)

1.25 Use GUI module *g_sample* to plot the magnitude responses of the following anti-aliasing filters. What is the oversampling factor, α, in each case?

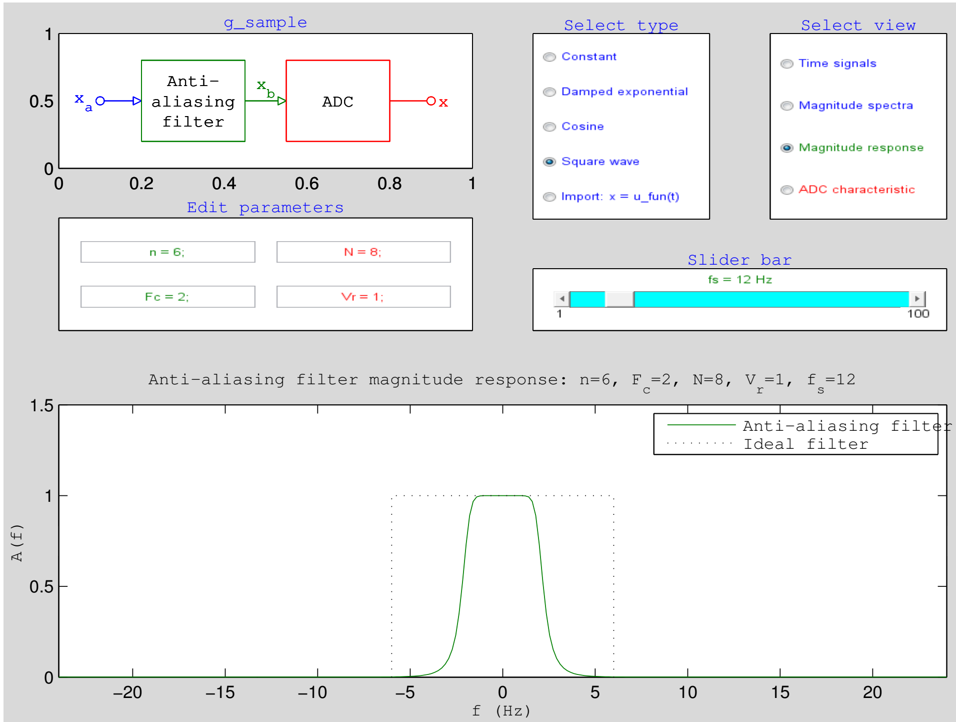
(a) $n = 2,\ F_c = 1,\ f_s = 2$

(b) $n = 6,\ F_c = 2,\ f_s = 12$

Solution



Problem 1.25 (a) Oversampling factor: $\alpha = f_s/(2F_c) = 1$



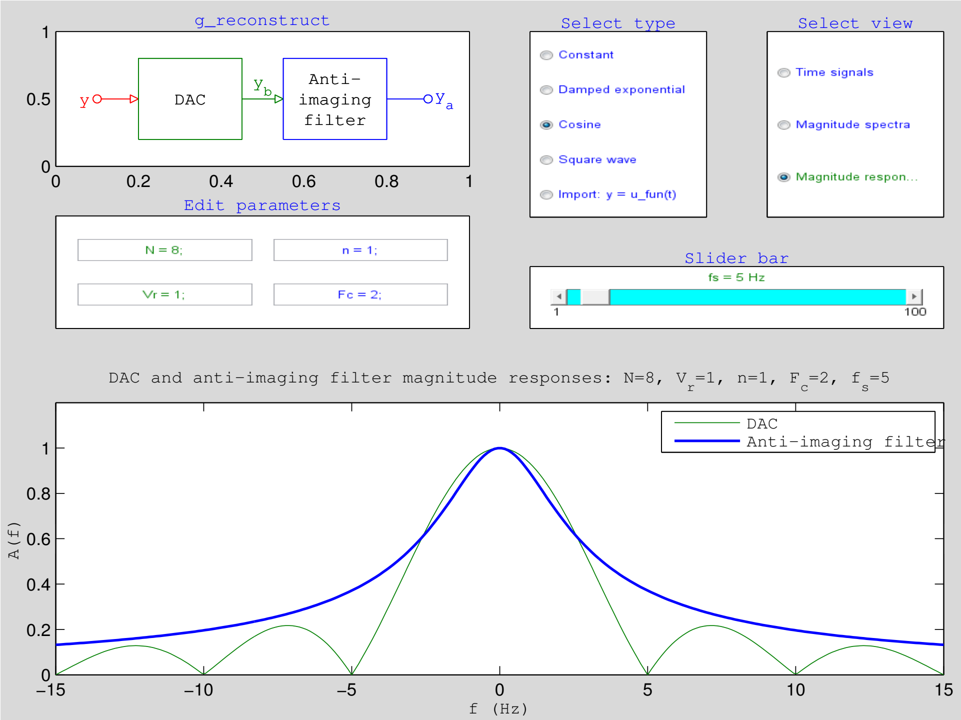
Problem 1.25 (b) Oversampling factor: $\alpha = f_s/(2F_c) = 3$

1.26 Use GUI module *g_reconstruct* to plot the magnitude responses of the following anti-imaging filters. What is the oversampling factor in each case?

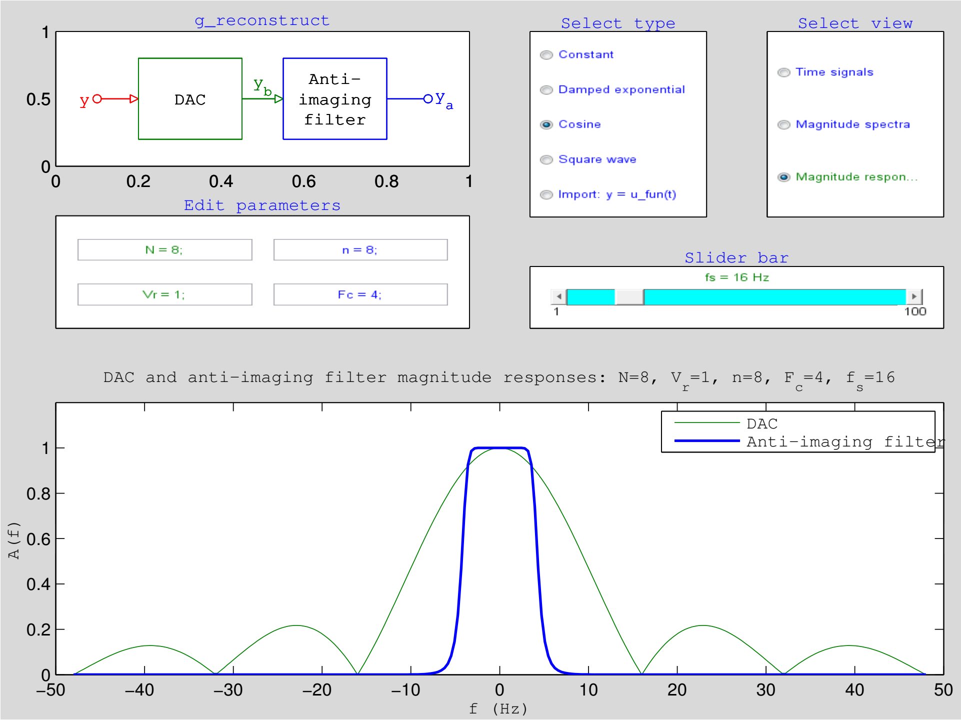
(a) $n = 1,\ Fc = 2,\ f_s = 5$

(a) $n = 8,\ Fc = 4,\ f_s = 16$

Solution



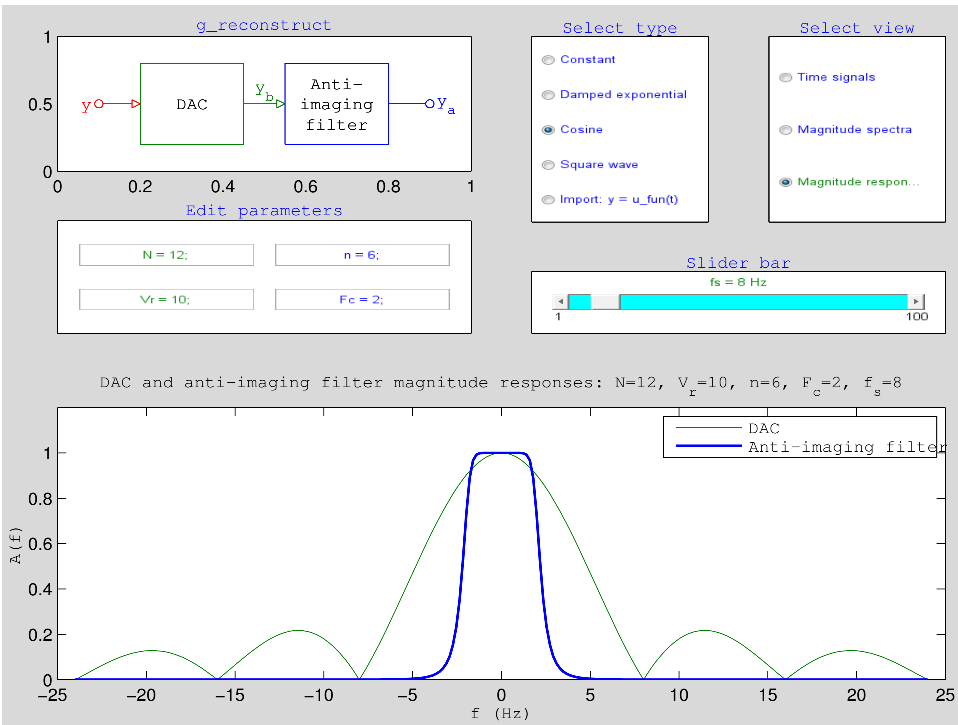
Problem 1.26 (a) Oversampling factor: $\alpha = f_s/(2F_c) = 1.25$



Problem 1.26 (b) Oversampling factor: $\alpha = f_s/(2F_c) = 2$

$\boxed{1.27}$ Use the GUI module *g_reconstruct* to plot the magnitude response of a 12-bit DAC with reference voltage $V_r = 10$ volts and a 6th order Butterworth anti-imaging filter with cutoff frequency $F_c = 2$ Hz. Use oversampling by a factor of two.

Solution



Problem 1.27

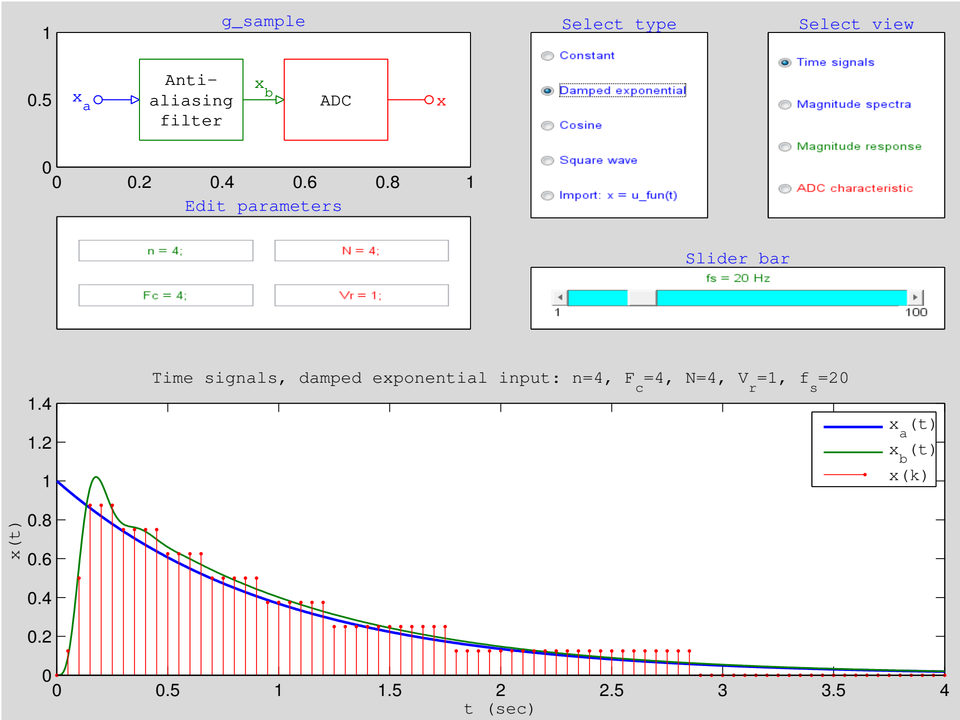
$\boxed{1.28}$ Use GUI module *g_sample* with the damped exponential input to plot the time signals using the following ADCs. For what cases does the ADC output saturate? Write down the quantization level on each time plot.

(a) $N = 4$, $V_r = 1$

(b) $N = 8$, $V_r = .5$

(c) $N = 8$, $V_r = 1$

Solution



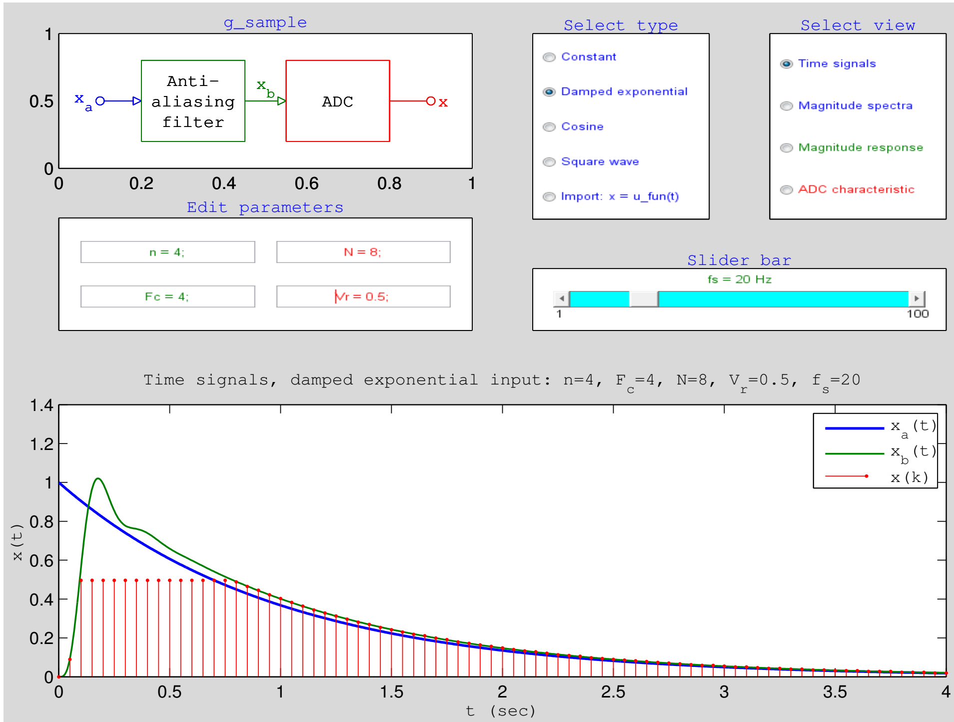
Problem 1.28 (a) No saturation, $q = 1/8$

1.29 Use GUI module *g_reconstruct* with the damped exponential input to plot the time signals using the following DACs. What is the quantization level in each case?

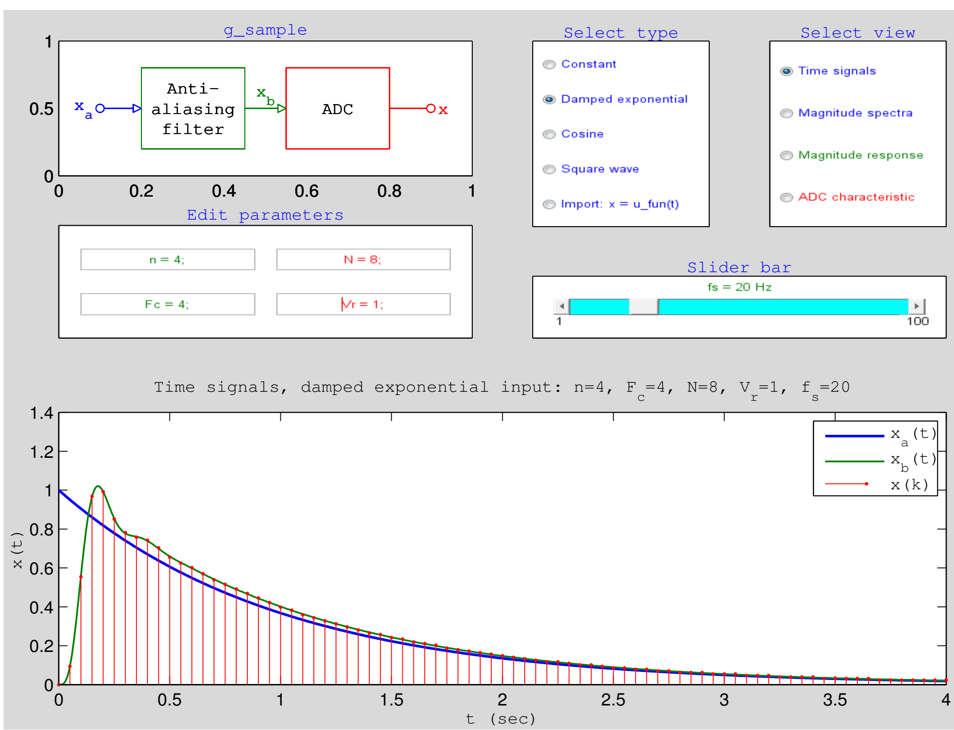
(a) $N = 4$, $V_r = .5$

(b) $N = 12$, $V_r = 2$

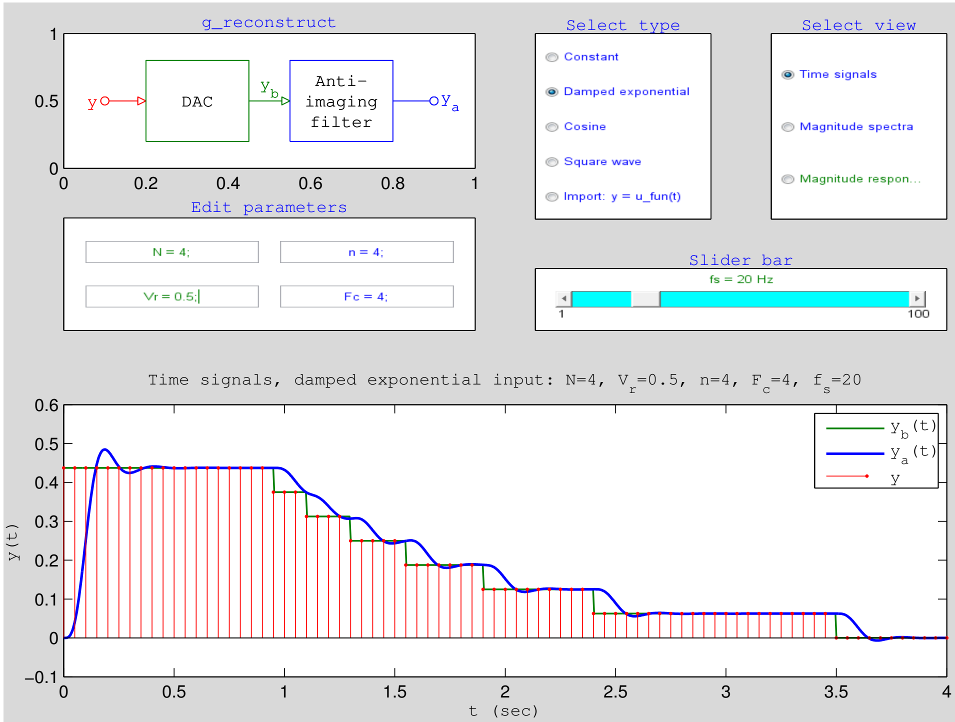
Solution



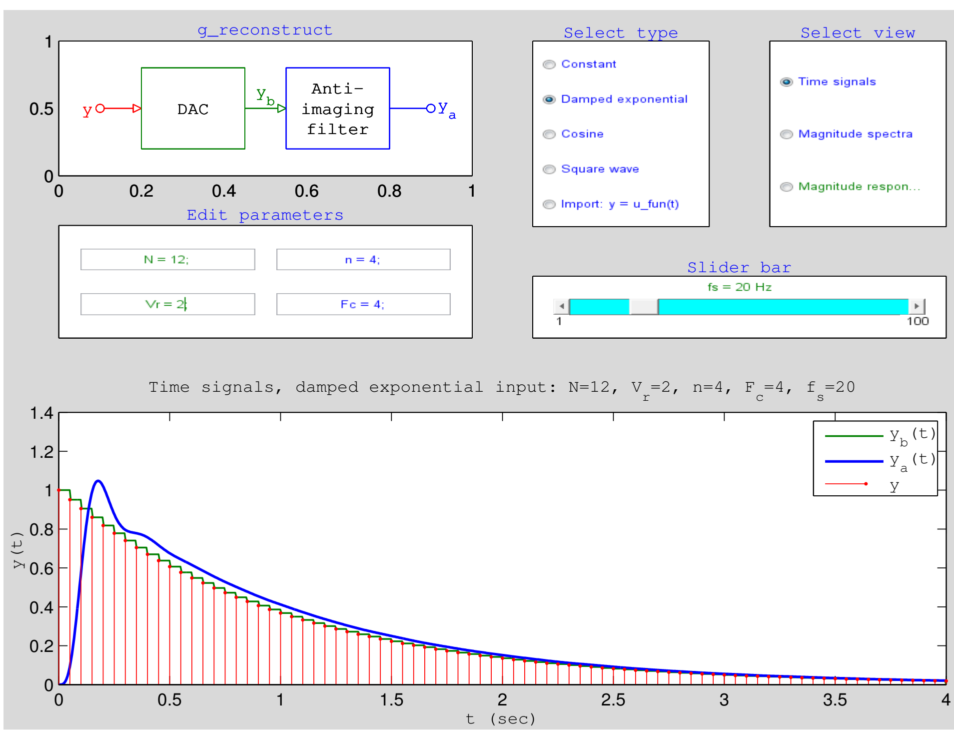
Problem 1.28 (b) Saturation at 0.5, $q = 1/256$



Problem 1.28 (c) No saturation, $q = 1/256$



Problem 1.29 (a) $q = 1/16$



Problem 1.29 (b) $q = 1/1024$

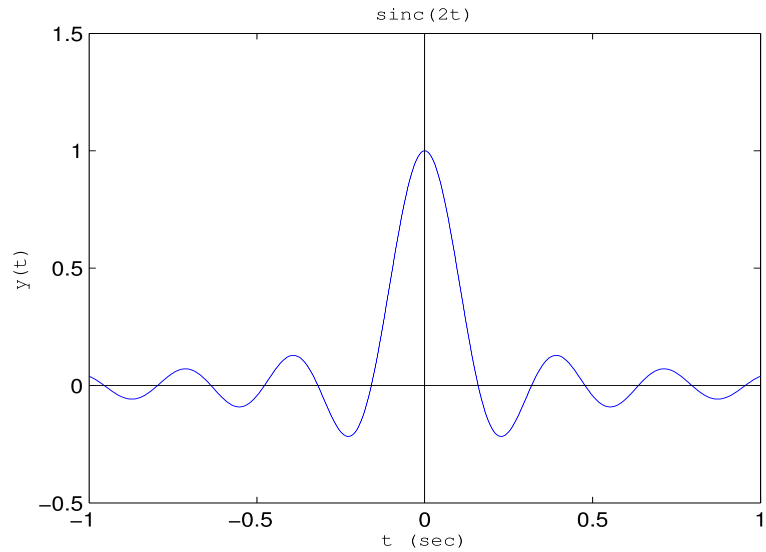
1.30 Write a MATLAB function called *u_sinc* that returns the value of the sinc function

$$\text{sinc}(x) \quad = \quad \frac{\sin(x)}{x}$$

Note that, by L'Hospital's rule, $\text{sinc}(0) = 1$. Make sure your function works properly when $x = 0$. Plot $\text{sinc}(2\pi t)$ for $-1 \le t \le 1$ using $N = 401$ samples.

Solution

```
% Problem 1.30

f_header('Problem 1.30')
p = 401;
t = linspace (-1,1,p);
y = u_sinc(2*pi*t);
figure
plot (t,y)
f_labels ('sinc(2t)','t (sec)','y(t)')
set (gca,'FontSize',11)
hold on
plot([-1 1],[0 0],'k')
plot([0 0],[-0.5 1.5],'k')
f_wait

function y = u_sinc (x)

% U_SINC: Implement the sifting function sin(pi*x)/(pi*x)
%
% Usage: y = u_sinc (x);
%
% Inputs: x = input scalar or vector
%
% Outputs: y = sin(pi*x)/(pi*x)

for i = 1 : length(x)
    if abs(x(i)) < eps
        y(i) = 1;
    else
        y(i) = sin(pi*x(i))/(pi*x(i));
    end
end
```



Problem 1.30 Sinc Function

1.31 The purpose of this problem is to numerically verify the signal reconstruction formula in Proposition 1.2. Consider the following bandlimited periodic signal, which can be thought of as a truncated Fourier series.

$$x_a(t) \quad = \quad 1 - 2\sin(\pi t) + \cos(2\pi t) + 3\cos(3\pi t)$$

Write a MATLAB script which uses the function *u_sinc* from problem 1.30 to approximately reconstruct $x_a(t)$ as follows.

$$x_p(t) \quad = \quad \sum_{k=-p}^{p} x_a(kT)\text{sinc}[f_s(t-kT)]$$

Use a sampling rate of $f_s = 6$ Hz. Plot $x_a(t)$ and $x_p(t)$ on the same graph using 101 points equally spaced over the interval $[-2, 2]$. Using *f_prompt*, prompt for the number p and do the following three cases.

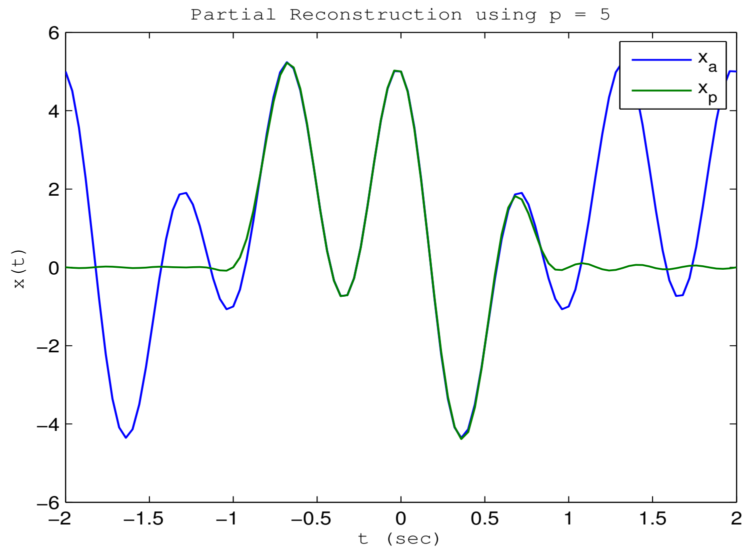
(a) $p = 5$

(b) $p = 10$

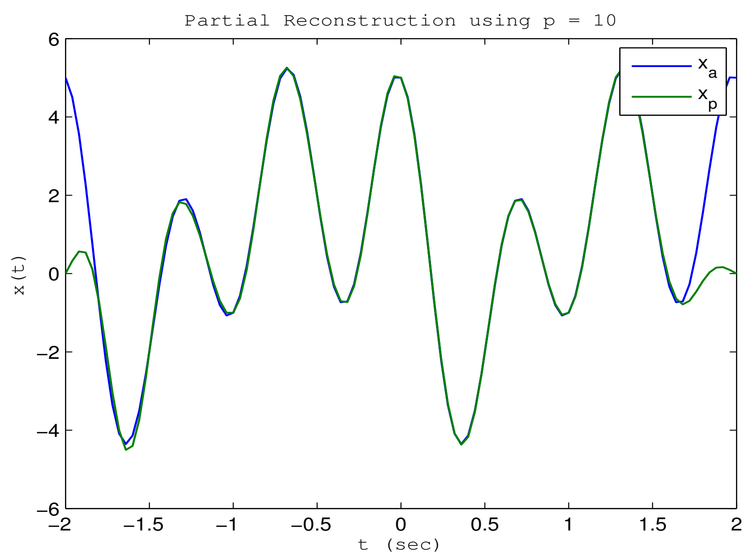
(c) $p = 20$;

Solution

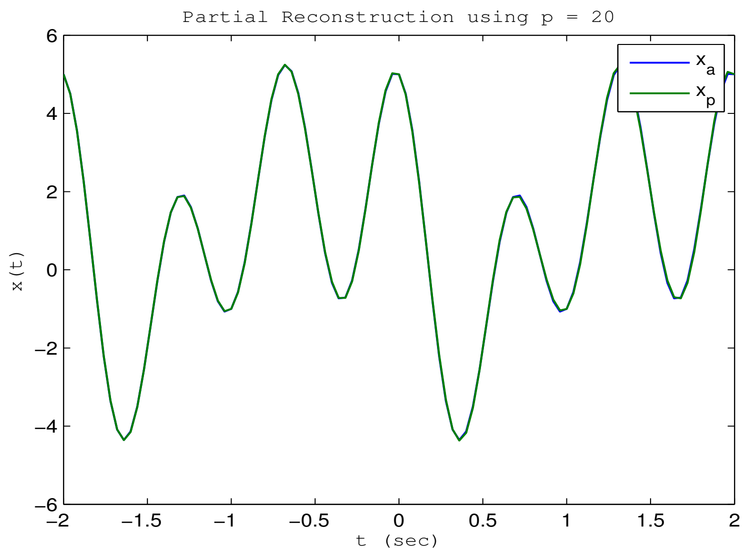
```
% Problem 1.31

% Initialize

f_header('Problem1.31')
x_a = inline ('1-2*sin(pi*t)+cos(2*pi*t)+3*cos(3*pi*t)','t');
fs = 6;
T = 1/fs;

% Reconstruct x_a(t) from it samples

p = f_prompt ('Enter number of terms p',0,40,10);
t = linspace (-2,2,101);
x_p = zeros(size(t));
for i = 1 : length(t)
   for k = -p : p
      x_p(i) = x_p(i) + x_a(k*T)*u_sinc(fs*(t(i) - k*T));
   end
end
figure
plot (t,x_a(t),t,x_p,'LineWidth',1.0)
caption = sprintf ('Partial Reconstruction using p = %d',p);
f_labels (caption,'t (sec)','x(t)')
legend ('x_a','x_p')
f_wait
```



Problem 1.31 (a)



Problem 1.31 (b)

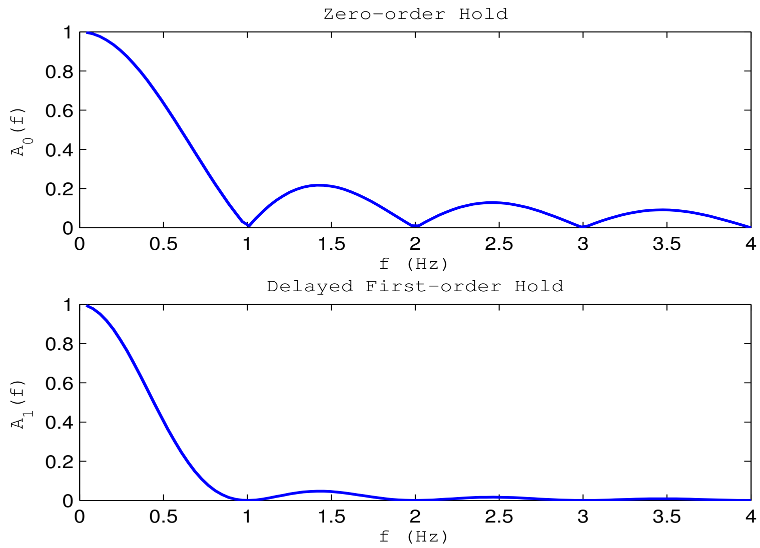


Problem 1.31 (c)

1.32 Consider the zero-order hold and delayed first-order hold filters used to reconstruct continuous-time signals from their samples. Since both filters are causal, the frequency responses can be obtained from the transfer functions by replacing s by $j2\pi f$ in (1.4.9) and (1.4.15), respectively. Write a MATLAB script that computes and plots the magnitude response of each filter. Use $f_s = 1$ Hz and plot both magnitude responses over $[0, 4]$ Hz on the same graph using a legend.

Solution

```
% Problem 1.32

% Initialize

f_header('Problem 1.32')
N =100;
fs = 1;
T = 1/fs;
fmax = 4;

% Find magnitude responses

f = linspace(0,fmax,N);
j = sqrt(-1);
s = j*2*pi*f;
H_0 = (1 - exp(-T*s)) ./ s;
A_0 = abs(H_0);
H_1 = ((1 - exp(-T*s)) ./ (T*s)).^2;
A_1 = abs(H_1);

% Plot them

figure
subplot(2,1,1);
plot(f,A_0,'LineWidth',1.5);
f_labels('Zero-order Hold','f (Hz)','A_0(f)')
subplot(2,1,2)
plot(f,A_1,'LineWidth',1.5);
f_labels('Delayed First-order Hold','f (Hz)','A_1(f)')
f_wait
```



Problem 1.32 Zero-order and Delayed First-order Holds

1.33 The Butterworth filter is optimal in the sense that, for a given filter order, the magnitude response is as flat as possible in the passband. If ripples are allowed in the passband, then an analog filter with a sharper cutoff can be achieved. Consider the following Chebyshev I lowpass filter from Chapter 7.

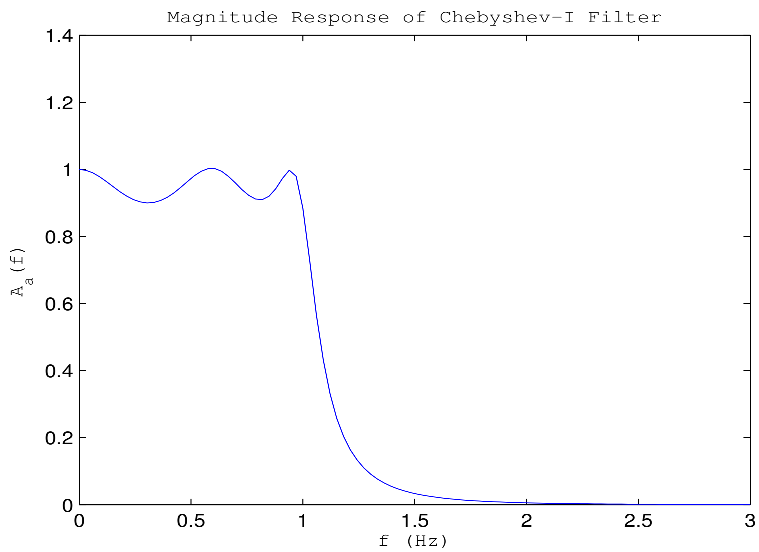
$$H_a(s) = \frac{1263.7}{s^5 + 6.1s^4 + 67.8s^3 + 251.5s^2 + 934.3s + 1263.7}$$

Write a MATLAB script the uses the DSP Companion function *f_freqs* to compute the magnitude response of this filter. Plot it over the range $[0, 3]$ Hz. This filter is optimal in the sense that the passband ripples are all of the same size.

Solution

```
% Problem 1.33

% Initialize

f_header('Problem 1.33')
N =100;
fmax = 3;
b = 1263.7
a = [1 6.1 67.8 251.5 934.3 1263.7]
```

```
% Compute and plot magnitude response

[H_a,f] = f_freqs (b,a,N,fmax);
A_a = abs(H_a);
figure
plot (f,A_a)
f_labels ('Magnitude Response of Chebyshev-I Filter','f (Hz)','A_a(f)')
axis([0 3 0 1.4])
f_wait
```



Problem 1.33 Chebyshev-I Filter

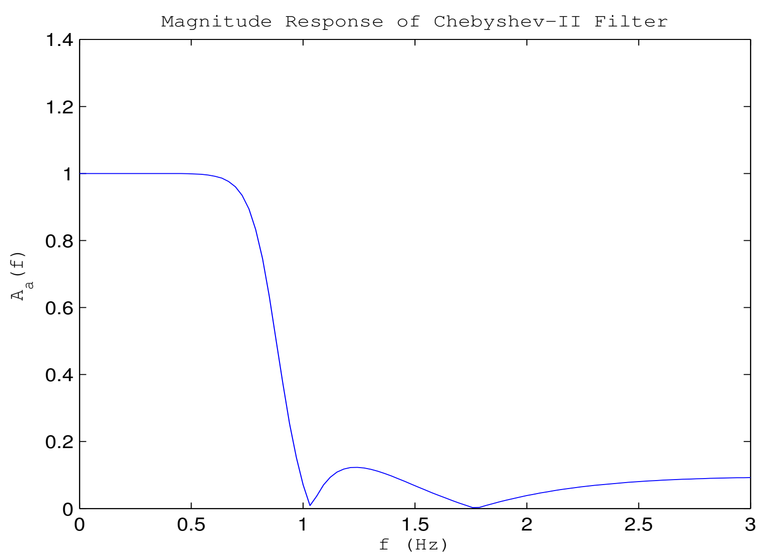
√ 1.34 Consider the following Chebyshev II lowpass filter from Chapter 7.

$$H_a(s) = \frac{3s^4 + 499s^2 + 15747}{s^5 + 20s^4 + 203s^3 + 1341s^2 + 5150s + 15747}$$

Write a MATLAB script the uses the DSP Companion function *f_freqs* to compute the magnitude response of this filter. Plot it over the range $[0, 3]$ Hz. This filter is optimal in the sense that the stopband ripples are all of the same size.

Solution

```
% Problem 1.34

% Initialize

f_header('Problem 1.34')
N =100;
fmax = 3;
b = [3 0 499 0 15747]
a = [1 20 203 1341 5150 15747]

% Compute and plot magnitude response

[H_a,f] = f_freqs (b,a,N,fmax);
A_a = abs(H_a);
figure
plot (f,A_a)
f_labels ('Magnitude Response of Chebyshev-II Filter','f (Hz)','A_a(f)')
axis([0 3 0 1.4])
f_wait
```



Problem 1.34 Chebyshev-II Filter

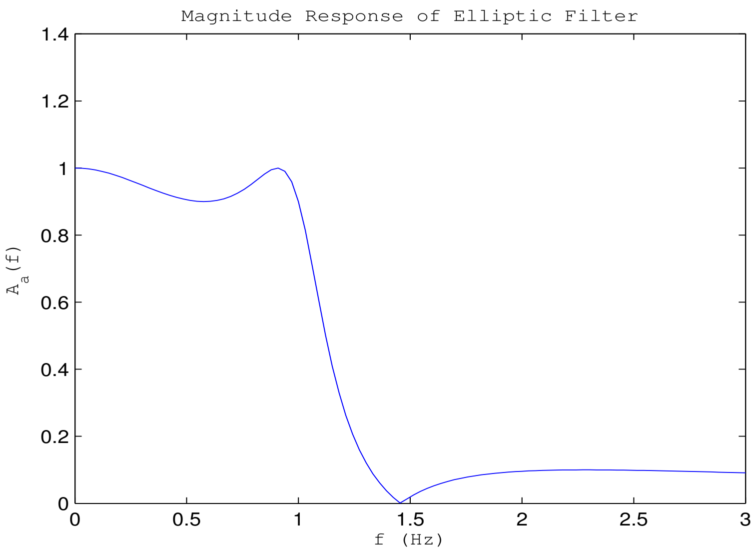
1.35 Consider the following elliptic lowpass filter from Chapter 7.

$$H_a(s) = \frac{2.0484s^2 + 171.6597}{s^3 + 6.2717s^2 + 50.0487s + 171.6597}$$

Write a MATLAB script the uses the DSP Companion function *f_freqs* to compute the magnitude response of this filter. Plot it over the range $[0, 3]$ Hz. This filter is optimal in the sense that the passband ripples and the stopband ripples are all of the same size.

Solution

```
% Problem 1.35

% Initialize

f_header('Problem 1.35')
N =100;
fmax = 3;
b = [2.0484 0 171.6597]
a = [1 6.2717 50.0487 171.6597]

% Compute and plot magnitude response

[H_a,f] = f_freqs (b,a,N,fmax);
A_a = abs(H_a);
figure
plot (f,A_a)
f_labels ('Magnitude Response of Elliptic Filter','f (Hz)','A_a(f)')
axis([0 3 0 1.4])
f_wait
```



Problem 1.35 Elliptic Filter